Serial Turbo Trellis Coded Modulation with Rate-1 Inner Code

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Abstract

The main objective of this paper is to develop new, low complexity turbo codes suitable for bandwidth and power limited systems, for very low bit and word error rate requirements. Motivated by the structure of recently discovered low complexity codes such as Repeat-Accumulate (RA) codes with low density parity check matrix, we extend the structure to high-level modulations such as 8PSK, and 16QAM. The structure consists of a simple 4-state convolutional or short block code as an outer code, and a rate-1, 2 or 4-state inner code. The inner code and the mapping are jointly optimized based on maximizing the effective free Euclidean distance of the inner TCM.

1 Introduction

Trellis coded modulation (TCM) proposed by Ungerboeck in 1982 [1] is now a well-established technique in digital communications. Since its first appearance, TCM has generated a continuously growing interest, concerning its theoretical foundations as well as its numerous applications, spanning high-rate digital transmission over voice circuits, digital microwave radio relay links, and satellite communications. In essence, it is a technique to obtain significant coding gains (3-6 dB) sacrificing neither data rate nor bandwidth.

Turbo codes represent a more recent development in the coding research field [2], which has raised a large interest in the coding community. They are *parallel concatenated convolutional codes* (PCCC) whose encoder is formed by two (or more) *constituent* systematic encoders joined through one or more interleavers. The input information bits feed the first encoder and, after having been scrambled by the interleaver, enter the second encoder. A codeword of a parallel concatenated code consists of the input bits to the first encoder followed by the parity check bits of both encoders.

The suboptimal iterative decoding structure is modular, and consists of a set of concatenated decoding modules, one for each constituent code, connected through the same interleaver used at the encoder side. Each decoder performs weighted soft decoding of the input sequence. Parallel concatenated convolutional codes yield very large coding gains at the expense of a data rate reduction, or bandwidth increase. In [4] we merged TCM and PCCC in order to obtain large coding gains and high bandwidth efficiency.

For certain applications, we require very low bit error rates (10^{-9}). To achieve this goal we suggest to merge TCM with the recently discovered serial concatenated codes (SCCC) [5], which have lower error floors, and adapting the concept of iterative decoding used in parallel concatenated codes. We note that the proposed serial concatenated coding scheme differs from "classical" concatenated coding systems. In the classical scheme the role of the interleaver between the two encoders is just to separate bursts of errors produced by the inner decoder, and no attempt is made to consider the combination of the two encoders and the interleaver as a single entity. Thus, the idea behind the recent serial schemes is new since we want to construct and optimize the whole serial structure. No such attempt was made in the past for two reasons. First, optimizing the overall code with large deterministic interleavers was prohibitively complex. However, by introducing the concept of *uniform interleaver* [6] it is possible to draw some criteria to optimize the component codes for the construction of powerful serial concatenated codes with large block size. Second, optimum decoding of such complex codes is practically impossible. Only the use of suboptimum iterative decoding methods makes it possible to decode such complex codes. In the following, we will call the concatenation of an outer convolutional or a short block code with an inner TCM a *serially concatenated TCM*

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(SCTCM). For parallel concatenated trellis coded modulation (PCTCM), also addressed as "turbo TCM", a first attempt employing the so-called "pragmatic" approach to TCM was described in [7]. Later, turbo codes were embedded in multilevel codes with multistage decoding [8]. Recently, punctured versions of Ungerboeck codes were used to construct turbo codes for 8PSK modulation [9]. In [4] we proposed a new solution to PCTCM with multilevel amplitude/phase modulations, and a suitable bit-by-bit iterative decoding structure. Preliminary results [4] showed that the performance of the proposed codes is within 1 dB from the Shannon limit at bit error probabilities of 10^{-7} for large block sizes. Unfortunately PCTCM [4] and all other proposed schemes in [7], [8] and [9] may produce an error floor above 10^{-9}. However, SCTCM is expected to have much lower error floor.

2 Serial Concatenated Trellis Coded Modulation: Code Design

The basic structure of serially concatenated trellis coded modulation was proposed in [13] and is shown in Fig. 1. We developed a method to design serial concatenated TCM, which achieves b bits/sec/Hz, using a rate $2b/(2b+1)$ binary convolutional encoder (or a short block code) with maximum free Hamming distance (or minimum distance) as the outer code. An interleaver π permutes the output of the outer code. The interleaved data enters a rate $(2b+1)/(2b+2)$ recursive convolutional inner encoder. The $2b+2$ output bits are then mapped to two symbols each belonging to a 2^{b+1} level modulation (four dimensional modulation). In this way, we are using $2b$ information bits for every two modulation symbol intervals, resulting in b bit/sec/Hz transmission (when ideal Nyquist pulse shaping is used) or, in other words, b bits per modulation symbol. The inner code and the mapping are jointly optimized based on maximizing the effective free Euclidean distance of the inner TCM.

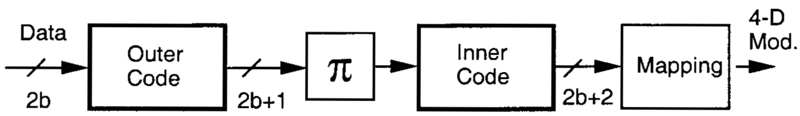

Figure 1: Block Diagram of the Encoder for Serial Concatenated Trellis Coded Modulation.

Since the invention of TCM by Ungerboeck in 1982, there have been numerous papers on the design of two- and multi-dimensional TCM. Unfortunately, we cannot use the conventional TCM designs for serial or parallel concatenated TCM, even if the structure of the encoder has a feedback as for conventional TCM. There are two main reasons:

1. The first condition to be satisfied for the inner encoder in serial TCM is that the Euclidean distance of encoded sequences be very large for input sequences having Hamming distance equal to 1. This may not be satisfied even if the encoder structure of conventional TCM has feedback. In conventional TCM, in fact, part of the input bits remain uncoded. These bits select a point from a subconstellation (coset) which, in turn, has been chosen according to the encoded bits. The combination of coded and uncoded bits is mapped to two or higher dimensional modulation. One could think of using conventional TCM without parallel branches, but this requires that the number of states be greater than the number of transitions per state, and this, in turn, prevents the use of simple codes with small number of states.

2. For the design of conventional TCM, the assignment of input labels does not play an important role, since it has a small impact on the bit error probability. As a consequence, the input labels assignment was typically arbitrary. For the design of SCTCM (and also for PCTCM), on the opposite, the input label assignment is crucial, as we will see in the following.

2.1 Rationale for Low-Complexity Code Selection

Although the above SCTCM structure results in a powerful code, we note that the number of transitions per state for the inner TCM is 2^{2b+1}. For the case of interest we have $b = 3$. Thus even if we keep the number of states low say 2, we have 128 transitions per state, which results in 256 edges in the trellis section. The complexity of the decoder depends on the number of edges per trellis section. Therefore for high speed operation we can't afford such complexity.

Our recent results on concatenation of an outer code with a simple accumulator as inner code for binary modulation [15] [14] led us to develop a second method for serial concatenated TCM (SCTCM). For MPSK, or a two dimensional constellation with M points, let's define $m = \log_2 M$, where M is number of phases. We propose a novel method, with lower complexity, to design serial concatenated TCM, which achieves $bm/(b+1)$ bits/sec/Hz, using a rate $b/(b+1)$ binary convolutional encoder (or a short block code) with maximum free Hamming distance (or minimum distance) as the outer code. An interleaver π permutes the output of the outer code. The interleaved data enters a rate $m/m = 1$ recursive convolutional inner encoder. The m output bits are then mapped to one symbol belonging to a 2^m level modulation. The structure of the SCTCM encoder is shown in Fig. 2.

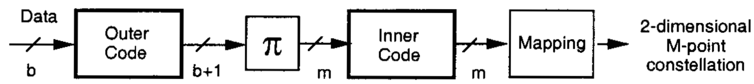

Figure 2: Structure of the encoder for serial concatenated trellis coded modulation. (2-D, M-point constellation).

In this way, we are using b information bits per $\frac{b+1}{m}$ modulation symbol interval, resulting in $bm/(b+1)$ bit/sec/Hz transmission (when ideal Nyquist pulse shaping is used) or, in other words, $bm/(b+1)$ bits per modulation symbol. The inner code and the mapping are jointly optimized based on maximizing the effective free Euclidean distance of the inner TCM. For example consider 8PSK modulation, where $m = 3$, then the throughput $r = 3b/(b+1)$ is as follows: for $b = 2$, $r = 2$; for $b = 3$, $r = 2.25$; and for $b = 4$, $r = 2.4$. This suggest that we can use a rate 1/2 convolutional code with puncturing to obtain various throughputs without changing the inner code or modulation.

For rectangular M^2-QAM, where $m = \log_2 M$, the structure becomes even simpler. In this case, to achieve throughput of $2mb/(b+1)$ bps/Hz we need a rate $b/(b+1)$ outer code and a rate m/m inner code, where the m output bits are alternatively assigned to in-phase and quadrature components of the M^2-QAM modulation. The structure of the SCTCM encoder is shown in Fig. 3. For example consider 16-QAM modulation, where $m = 2$, then the throughput $r = 4b/(b+1)$ is: for $b = 1, r = 2$; for $b = 2, r = 2.67$; for $b = 3, r = 3$; and for $b = 4, r = 3.2$.

For the case of interest we have $b = r = 3$. We note that now the number of transitions per state of the inner TCM is reduced to 4 (this results in a large reduction in complexity: 32 times lower than the previous case). Moreover, the outer code also has lower code rate (from 6/7 to 3/4). Here we only consider the example of 16QAM modulation, and $r = 3$ which implies $b = 3$. The encoder structure of SCTCM for 2-state inner TCM is shown in Fig. 4. The encoder structure of SCTCM for 4-state inner TCM is shown in Fig. 5.

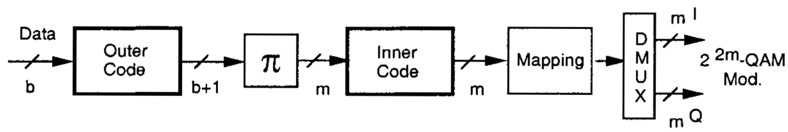

Figure 3: Structure of the encoder for serial concatenated trellis coded modulation. (M^2 QAM).

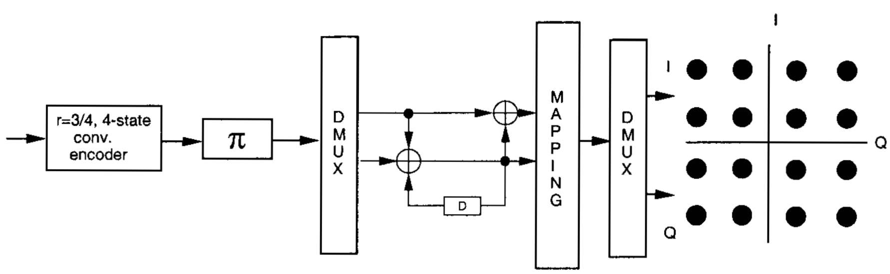

Figure 4: 3 bps/Hz Turbo Trellis Coded Modulation with 2-state inner TCM

The output of inner encoder in each case is mapped to the I and Q components of 16QAM alternatively . The outer code is an optimum rate 3/4, 4-state nonrecursive convolutional code with free Hamming distance of 3. The structure of outer encoder is shown in Fig. 6.

The optimum rate 3/4, 4-state outer code has 32 edges per trellis section and produces 4 output bits. Thus the complexity per output bit is 32/4=8. The complexity per input bit is 32/3. To further reduce the complexity of the

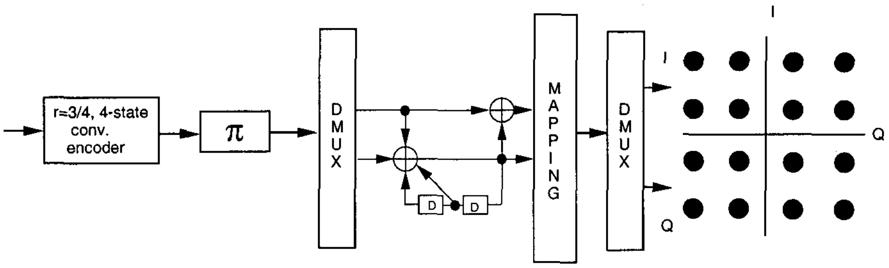

Figure 5: 3 bps/Hz Turbo Trellis Coded Modulation with 4-state inner TCM

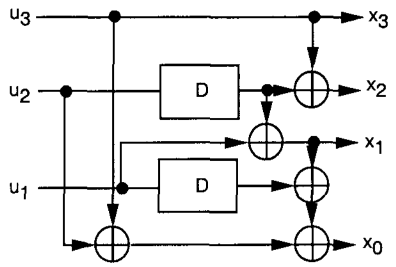

Figure 6: Optimum rate 3/4, 4-state outer code

outer code we suggest to use a rate 1/2, 4-state systematic recursive convolutional code. This code can be punctured to rate 3/4, by puncturing only the parity bits. The minimum distance of this punctured code is 3, the same as for optimum code. Now the code has 8 edges per trellis section and produces 2 output bits. Thus the complexity per output bit is 8/2=4. Since this code is systematic there is no complexity associated with the input bit (see description of SISO for outer code in iterative decoding). The encoder structure for this low complexity SCTCM is shown in Figure 7. If we use the proposed low complexity SCTCM with 4-state outer and 4-state inner, with no correction terms in the SISO module, then the complexity of the proposed scheme with 5 iterations will be roughly equal to the complexity of a standard Viterbi decoder, but still obtaining a 2 dB advantage over the Pragmatic TCM system to be discussed later.

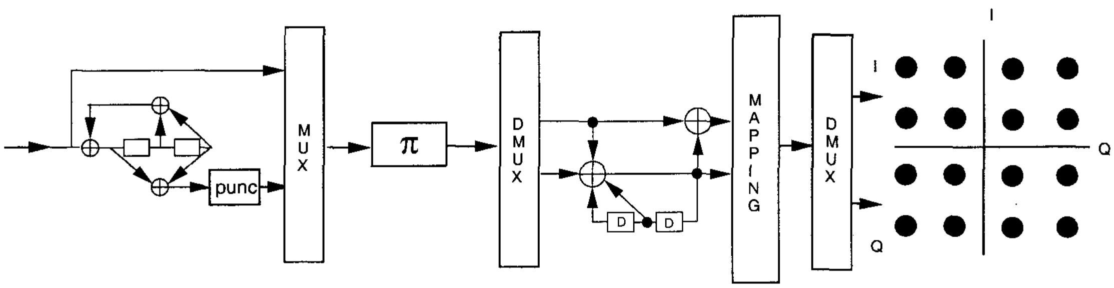

Figure 7: 3 bps/Hz low-complexity serial turbo trellis coded modulation

2.2 Design Criteria for Serially Concatenated TCM

It can be shown that the dominant term in the transfer function bound on bit error probability of serially concatenated TCM, employing an outer code with free (or minimum) Hamming distance d_f^o, averaged over all possible interleavers of size N bits, is proportional for large N to

$$N^{-\lfloor (d_f^o+1)/2 \rfloor} e^{-\delta^2 (E_s/4N_o)}$$

where $\lfloor x \rfloor$ represents the integer part of x, and

$$\delta^2 = \frac{d_f^o d_{\mathrm{f,eff}}^2}{2}, \quad \text{for } d_f^o \text{ even, and}$$

$$\delta^2 = \frac{(d_f^o - 3) d_{\mathrm{f,eff}}^2}{2} + (h_m^{(3)})^2, \quad \text{for } d_f^o \text{ odd.}$$

The parameter $d_{\mathrm{f,eff}}$ is the *effective free Euclidean distance* of the inner code (to be defined in the following), $h_m^{(3)}$ is the minimum Euclidean distance of inner code sequences generated by input sequences with Hamming distance 3, and E_s/N_o is the M-ary symbol signal-to-noise ratio.

The above results are valid for very large N. On the other hand, for large values of the signal-to-noise ratio E_s/N_o, the performance of SCTCM is dominated by

$$N^{-(l_m(h_m)-1)} e^{-h_m^2 (E_s/4N_o)}$$

where h_m is the minimum Euclidean distance of the SCTCM scheme, and $l_m(h_m)$ is the minimum Hamming distance between input sequences to the inner TCM encoder producing h_m. We note that $l_m(h_m) \geq d_f^o$.

Based on these results, the design criterion for serially concatenated TCM for large interleavers and very low bit error rates is to maximize the free Hamming distance of the outer code (to achieve interleaving gain), and to maximize the effective free Euclidean distance of the inner TCM code.

Let $\mathbf{z}$ be the binary input sequence to the inner TCM code, and $x(\mathbf{z})$ be the corresponding inner TCM encoder output with M-ary symbols. The criteria proposed for designing and selecting the constituent inner TCM encoder are the following:

1. Design the constituent inner TCM encoder for a given two or multidimensional modulation such that the minimum Euclidean distance $d(x(\mathbf{z}), x(\mathbf{z}'))$ over all $\mathbf{z}, \mathbf{z}'$ pairs, $\mathbf{z} \neq \mathbf{z}'$, is maximized, given that the Hamming distance $d_H(\mathbf{z}, \mathbf{z}') = 2$. We call this minimum Euclidean distance the *effective free Euclidean distance* of the inner TCM code and denote it simply by $d_{\mathrm{f,eff}}$.

2. If the free distance of the outer code d_f^o is odd, then, among the selected inner TCM encoders, choose those that have the maximum Euclidean distance $d(x(\mathbf{z}), x(\mathbf{z}'))$ over all $\mathbf{z}, \mathbf{z}'$ pairs, $\mathbf{z} \neq \mathbf{z}'$, given that the Hamming distance $d_H(\mathbf{z}, \mathbf{z}') = 3$. We call this the minimum Euclidean distance of the inner TCM code due to input Hamming distance 3, and denote it by $h_m^{(3)}$.

3. Among the candidate encoders, select the one that has the largest minimum Euclidean distance in encoded sequences produced by input sequences with Hamming distance d_f^o. We denote this minimum Euclidean distance of the SCTCM code by h_m.

One may ask why we care about sequences with Hamming distances of 2 or 3 at the input of the TCM encoder if the free Hamming distance d_f^o of the outer code is larger than 2 or even 3. The answer is that the interleaving gain at low SNR depends on the number of error events that a pair of input sequences generate in the trellis of the inner code. For a given input Hamming distance, the larger the number of error events is, the smaller the interleaving gain will be. Thus, for example, if the input Hamming distance between sequences to the inner TCM is 4, the largest number of error events that produce small output Euclidean distances is 2 (two events with an input Hamming distance of 2 each).

2.3 Mapping (output labels) for TCM

As soon as the input labels and output signals are assigned to the edges of a trellis we have a complete description of the TCM code. The selection of the mapping (output labels) does not change the trellis code. However, it influences the encoder circuit required to implement the TCM scheme. A convenient mapping should be selected to simplify the encoder circuit and, if possible, to yield a linear circuit that can be implemented with exclusive ORs. The set partitioning of the constellation and the assignment of constellation points to trellis edges, and the successive assignments of input labels to the edges are important. Ungerboeck [1] proposed a mapping called *mapping by set partitioning*, leading to the "natural mapping". This mapping for two-dimensional modulation is useful if one selects the TCM scheme by searching among all encoder circuits that maximize the minimum Euclidean distance.

2.4 Design Method for Inner TCM

The proposed design method is based on the following steps:

1. The well known set partitioning techniques for signal sets are used (see for example [10] and the references therein).

2. The input labels assignment is based on the codewords of the parity check code $(m, m-1, 2)$ and its set partitioning, to maximize the quantities described in subsection 2.2. Using this method the minimum Hamming distance between input labels for parallel transitions will be equal to 2. The assignment of codewords of the parity check code as input labels to the 2-dimensional signal points is not arbitrary.

3. A sufficient condition to have very large output Euclidean distances for input sequences with Hamming distance 1, is that all input labels to each state be distinct.

4. Assign pair of input labels and 2-dimensional signal points to the edges of a trellis diagram based on the design criteria in subsection 2.2.

2.5 Example of the Design Methodology

Example 1: Set partitioning of 8PSK and input labels assignment.
Let the eight phases of 8PSK be denoted by $\{0, 1, 2, 3, 4, 5, 6, 7\}$. Here $m = 3$. Consider the 8PSK signal set $A = [0, 2, 4, 6]$, and set $B = [1, 3, 5, 7]$. For unit radius 8PSK constellation, the minimum intra-set square Euclidean distance for each set is 2. The minimum inter-set square Euclidean distances 0.586 .

Select the input label set L_0 as codewords of the $(3, 2, 2)$ parity check code, i.e. $L_0 = [(000), (011), (101), (110)]$, next generate input label $L_1 = L_0 + (001)$, i.e., $L_1 = [(001), (010), (100), (111)]$. Consider a 2-state trellis. Assign the input-output pair (L_0, A) to four edges from state 0 to state 0. Assign the input-output pair (L_1, B) to four edges from state 0 to state 1. Next assign the input-output pair (L_2, A) to four edges from state 1 to state 0, and assign the input-output pair (L_3, B) to four edges from state 1 to state 1. L_2 has the same elements as in L_1 but with different order, and L_3 has the same elements as in L_0 again with different order. In order to maximize the minimum Euclidean distance due to the input sequences with Hamming distance 2, we have to find the right permutation within each set. In this case it turns out that using the complement operation suffices. Therefore define input label L_2 as the complement of the elements of L_1 without changing their order, i.e., $L_2 = [(111), (100), (010), (001)]$. Finally L_3 is generated in the same way, as the complement of the elements in L_0, i.e., $L_3 = [(110), (101), (011), (000)]$.

Such assignment guarantees that the squared effective free Euclidean distance of trellis code is 2, where the minimum squared Euclidean distance of the code is 0.586.
Having determined the code by its input labels and 2-dimensional output signals, the encoder structure can then be obtained by selecting any appropriate labels (output labels) for the 2-dimensional output signals. We used the following output mapping, [(000), (001), (010), (011), (110), (111), (100), (101)], mapped to phases [0, 1, 2, 3, 4, 5, 6, 7], which is called "reordered mapping". The Truth Table required to implement the code is shown in Table 1. The implementation of the 2-state inner trellis code was obtained from the Truth Table as shown in Fig. 8. For this 2-state inner code, $d^2_{\text{f,eff}} = 2$, $h^{(3)}_m = \infty$, and $h^2_m = 0.586$. The outer code for this example can been selected as an 4-state, rate 2/3, convolutional code with $d^o_f = 3$ (this is a recursive systematic rate 1/2 convolutional code where the parity bits are punctured). Since $h^{(3)}_m = \infty$ then d^o_f is increased effectively to 4. This method of design was used to obtain the encoders in the previous examples for 16QAM.

3 Performance Results

We considered the pragmatic TCM system shown in Fig. 9, as a reference for performance comparison.

It consists of an outer (255,239) Reed-Solomon (RS) encoder decoder, an inner convolutional (rate 1/2, k=7, $G1 = 133_{octal}$, $G2 = 171_{octal}$ encoder, and Viterbi decoder. The inner TCM is a rate 7/8, four-dimensional Pragmatic Trellis Coded Modulation (4D-PTCM), (See Fig. 10), with 16-ary Rectangular Quadrature Amplitude Modulation (QAM). The throughput of this system can be computed as:

Throughput = (#bits/symbol) * (inner code rate) * (outer code rate) = 4 bits/symbol * 7/8 * 239/255 = 3.28 bits/symbol.

The input frame size is $239 \times 8 \times 8 = 15296$, and the interleaver depth is 8×255.

P-State	Input Label	N-State	Phases	Output Label
p	b_2, b_1, b_0	n	ϕ	x_2, x_1, x_0
0	000	0	0	000
0	011	0	2	010
0	101	0	4	110
0	110	0	6	100
0	011	1	1	001
0	010	1	3	011
0	100	1	5	111
0	111	1	7	101
1	111	0	0	000
1	100	0	2	010
1	010	0	4	110
1	001	0	6	100
1	110	1	1	001
1	101	1	3	011
1	011	1	5	111
1	000	1	7	101

Table 1: Truth Table for the 2-state inner TCM with 8PSK.

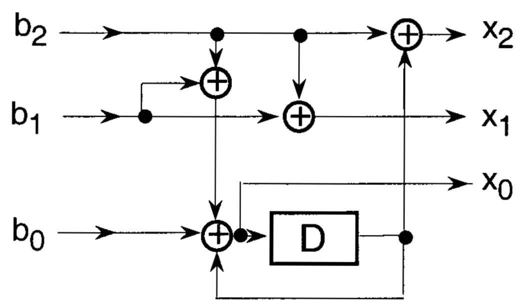

Figure 8: Optimum 2-state inner trellis encoder for SCTCM with 8PSK Modulation.

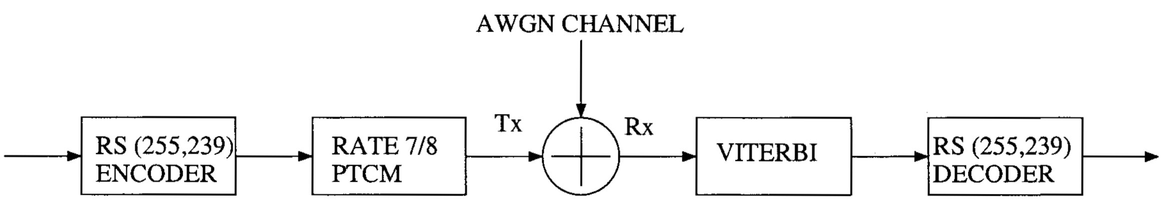

Figure 9: System Block Diagram

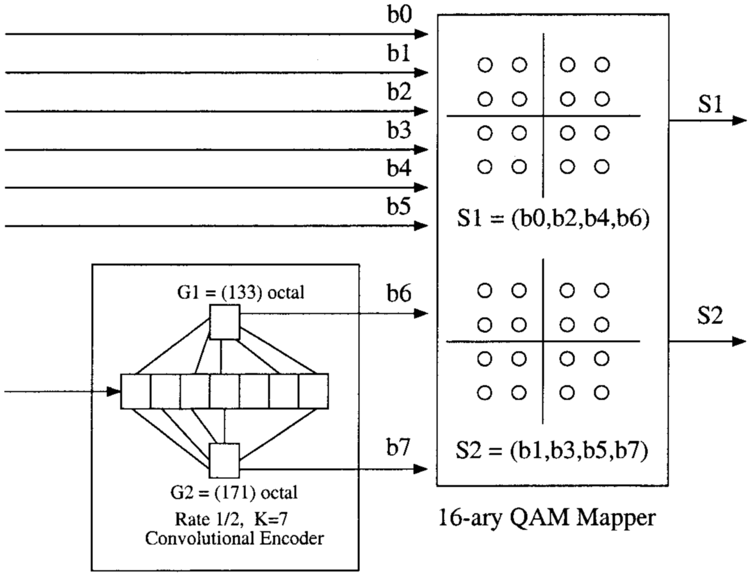

Figure 10: Rate 7/8 4D-PTCM

3.1 Simulation Results for PTCM

The simulations consist of a random data generator, concatenated rate 1/2, k=7 convolutional & Reed-Solomon (RS) (255,239) encoder, 16-ary QAM modulator, AWGN noise generator, 16-ary QAM demodulator and concatenated Viterbi & RS (255,239) decoder.

The four-dimensional Pragmatic Trellis Coded Modulation (4D-PTCM) scheme is designed for a high performance in a concatenated system and for ease of decoder implementation. The simulated performance of the rate 7/8 4D-PTCM with a Reed-Solomon (255,239) outer code is shown in Fig. 11. The concatenated coding scheme provides a throughput of 3.28 bits/symbol and requires ~9.0 dB to obtain a BER of 10^{-8} as shown in Fig. 11. In order to estimate the required E_b/N_o to achieve a BER of 10^{-9}, we extrapolated the simulation results and we obtained ~9.5 dB.

3.2 Simulation Results for serial TCM with rate 1 inner code

The iterative decoder performance of the proposed low complexity turbo serial TCM is shown in Fig. 12. In the simulations we used an optimum rate 3/4 outer code as shown in Fig. 6. Simulations for the punctured low complexity turbo serial TCM as shown in Fig. 7, for 2-state and 4-state inner TCM, are in progress.

The capacity of this signal set is shown in Fig. 13. As shown in Fig. 13, the capacity limit at throughput 3 bps/Hz, is E_b/N_o=4.54 dB, and at throughput 3.28 bps/Hz is E_b/N_o=5.36 dB. The Pragmatic TCM system designed for throughput 3.28 bps/Hz, at BER= 10^{-9} requires about 9.5 dB. Therefore the Pragmatic TCM system is 4.15 dB away from the capacity limit.

4 Iterative Decoder

4.1 Bit-by-Bit Iterative Decoding of Serially Concatenated Trellis Coded Modulation

The iterative decoder for serially concatenated trellis coded modulation uses a generalized Log-APP (a-posteriori probability) decoder module with four ports, called SISO APP module or simply SISO [11]. The block diagram of the iterative decoder for serial concatenated TCM is shown in Fig. 14.

We briefly describe the SISO algorithm for the inner TCM code and outer convolutional code, using the trellis section shown in Fig 15. Consider an inner TCM code with p_1 input bits and q_1 nonbinary complex output symbols with normalized unit power, and an outer code with p_2 input bits and q_2 binary outputs $\{0, 1\}$. Let $\mathbf{u}_k(e)$ represent

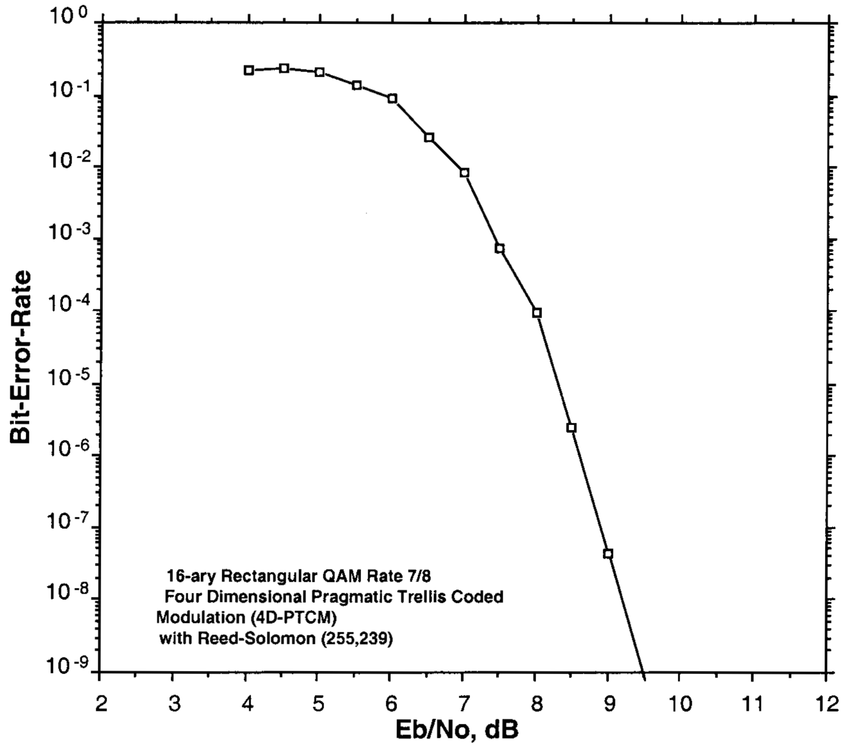

Figure 11: Performance of Pragmatic TCM system

$u_{k,i}(e)$; $i = 1, 2, \cdots, p_m$ the input bits on a trellis edge at time k ($m = 1$ for the inner TCM, and $m = 2$ for the outer code), and let $\mathbf{c}_k(e)$ represent $c_{k,i}(e)$; $i = 1, 2, \cdots, q_m$ the output symbols ($m = 1$ for the inner TCM, with nonbinary complex symbols, and $m = 2$ for the outer code with binary $\{0, 1\}$ symbols).

Define the reliability of a bit Z taking values $\{0, 1\}$ at time k as

$$\lambda_k[Z; \cdots] \triangleq \log \frac{P_k[Z = 1; \cdot]}{P_k[Z = 0; \cdot]}$$

The second argument in the brackets, shown as a dot, may represent I, the input, or O, the output, to the SISO. We use the following identity

$$a = \log[\sum_{i=1}^{L} e^{a_i}] = \max_i\{a_i\} + \delta(a_1, ..., a_L) \triangleq \max_i{}^*\{a_i\}$$

where $\delta(a_1, ..., a_L)$ is the correction term which can be computed using a look-up table. For more detail see [12] and its references number 25, 26, 31, and the reference in footnote 1. We define the "max*" operation as a maximization (compare/select) plus a correction term (lookup table). Small degradations occur if the "max*" operation is replaced by "max". The received complex samples $\{y_{k,i}\}$ at the output of the receiver matched filter are normalized such that additive complex noise samples have unit variance per dimension.

4.2 The SISO Algorithm for the Inner TCM

The *forward* and the *backward* recursions are:

$$\alpha_k(s) = \max_{e:s^E(e)=s}{}^* \{\alpha_{k-1}[s^S(e)] + \sum_{i=1}^{p_1} u_{k,i}(e)\lambda_k[U_{k,i}; I] + \sum_{i=1}^{q_1} \tilde{\lambda}_k[c_{k,i}(e); I]\} + h_{\alpha_k}$$

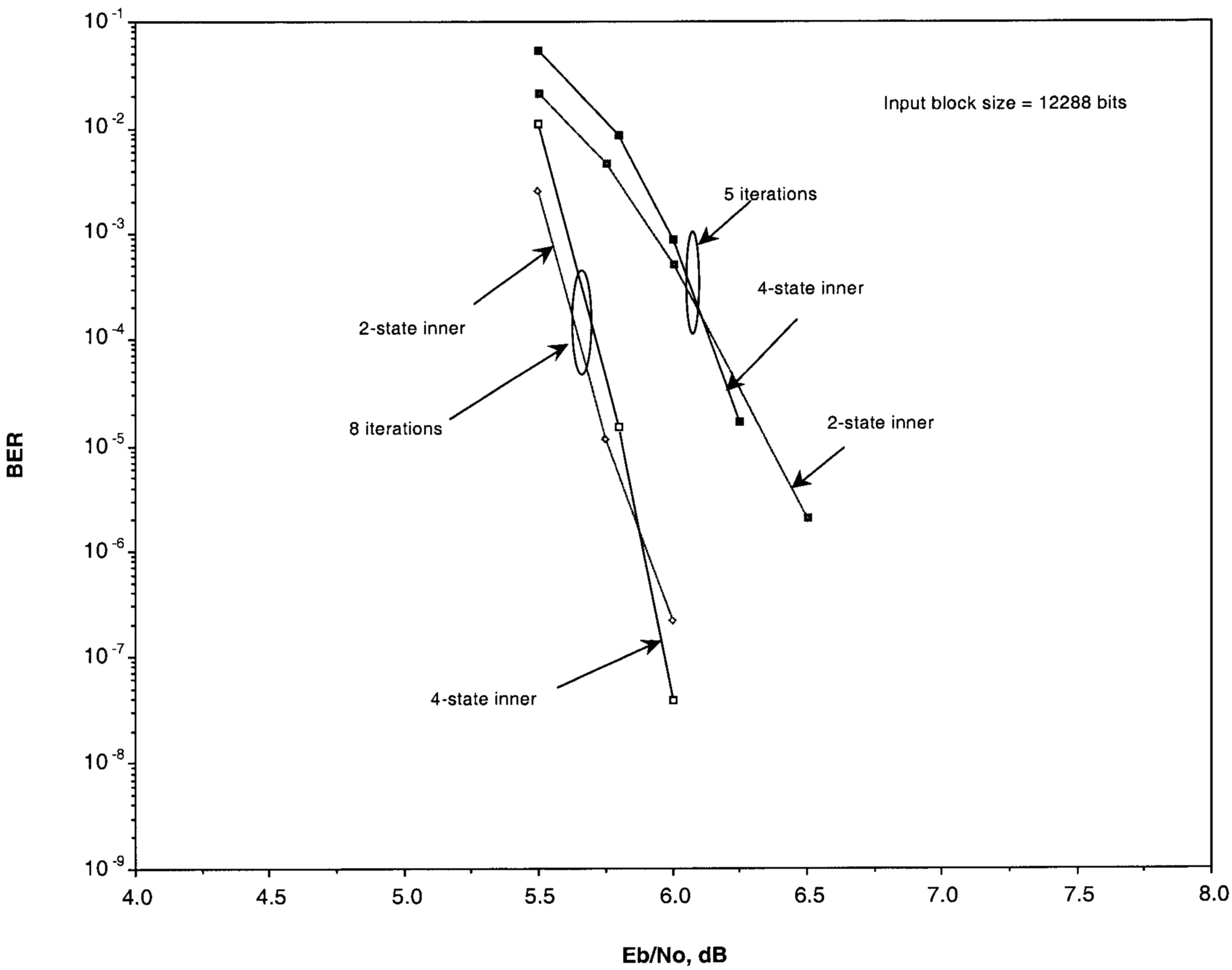

Figure 12: Performance of 3 bps/Hz Turbo Trellis Coded Modulation

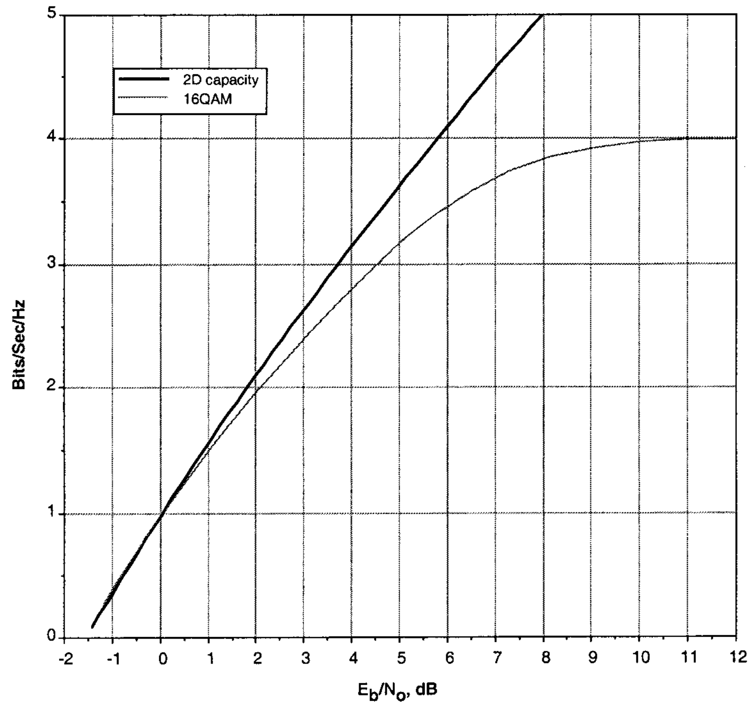

Figure 13: Capacity for 16QAM signaling

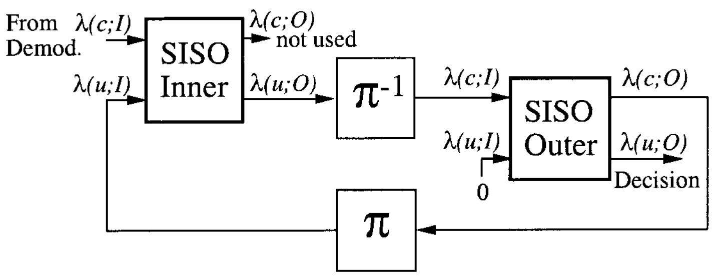

Figure 14: Iterative decoder for serially concatenated trellis coded modulation.

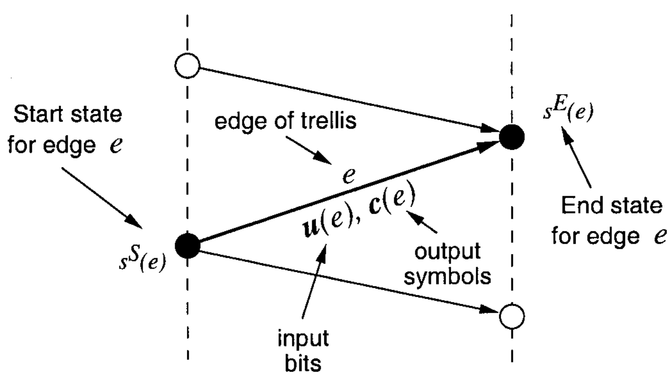

Figure 15: Trellis Section.

$$\beta_k(s) = \max_{e:s^S(e)=s}{}^* \{\beta_{k+1}[s^E(e)] + \sum_{i=1}^{p_1} u_{k+1,i}(e)\lambda_{k+1}[U_{k+1,i}; I] + \sum_{i=1}^{q_1} \tilde{\lambda}_{k+1}[c_{k+1,i}(e); I]\} + h_{\beta_k}$$

for all states s, and $k = 1, \ldots, (n-1)$, where n represents the total number of trellis steps from the initial state to the final state.
The *extrinsic bit information* for $U_{k,j}$; $j = 1, 2 \cdots, p_1$ can be obtained from:

$$\begin{aligned}\lambda_k(U_{k,j}; O) = &\max_{e:u_{k,j}(e)=1}{}^* \{\alpha_{k-1}[s^S(e)] + \sum_{\substack{i=1\\ i\neq j}}^{p_1} u_{k,i}(e)\lambda_k[U_{k,i}; I] + \sum_{i=1}^{q_1} \tilde{\lambda}_k[c_{k,i}(e); I] + \beta_k[s^E(e)]\}\\ &- \max_{e:u_{k,j}(e)=0}{}^* \{\alpha_{k-1}[s^S(e)] + \sum_{\substack{i=1\\ i\neq j}}^{p_1} u_{k,i}(e)\lambda_k[U_{k,i}; I] + \sum_{i=1}^{q_1} \tilde{\lambda}_k[c_{k,i}(e); I] + \beta_k[s^E(e)]\}\end{aligned}$$

where $\tilde{\lambda}_k[c_{k,i}(e); I] = -|y_{k,i} - \sqrt{\frac{2E_s}{N_o}} c_{k,i}(e)|^2/2$. We assume the initial and the final states of the inner encoder (as well as the outer encoder) are the all zero state. Forward recursions start with initial values, $\alpha_0(s) = 0$, if $s = 0$ (initial zero state) and $\alpha_0(s) = -\infty$, if $s \neq 0$. Backward recursions start with $\beta_n(s) = 0$, if $s = 0$ (final zero state) and $\beta_n(s) = -\infty$, if $s \neq 0$. The h_{α_k} and h_{β_k} are normalization constants which, in the hardware implementation of the SISO, are used to prevent buffer overflow. These operations are similar to the Viterbi algorithm used in the forward and backward directions, except for a correction term that is added when compare-select operations are performed. At the first iteration all $\lambda_k[U_{k,i}; I]$ are zero. After the first iteration, the inner SISO accepts the extrinsics from the outer SISO, through the interleaver π, as reliabilities of input bits of TCM encoder, and the external observations from the channel. The inner SISO uses the input reliabilities and observations for the calculation of new extrinsics $\lambda_k(U_{k,j}; O)$ for the input bits. These are then provided to the outer SISO module, through the deinterleaver π^{-1}.

4.3 The SISO Algorithm for the Outer Code

The *forward* and the *backward* recursions are:

$$\alpha_k(s) = \max_{e:s^E(e)=s}{}^* \{\alpha_{k-1}[s^S(e)] + \sum_{i=1}^{q_2} c_{k,i}(e)\lambda_k[C_{k,i}; I]\} + h_{\alpha_k}$$

$$\beta_k(s) = \max_{e:s^S(e)=s}{}^* \{\beta_{k+1}[s^E(e)] + \sum_{i=1}^{q_2} c_{k+1,i}(e)\lambda_k[C_{k+1,i}; I]\} + h_{\beta_k}$$

The *extrinsic information* for $C_{k,j}$; $j = 1, 2, \cdots, q_2$, can be obtained from:

$$\begin{aligned}\lambda_k(C_{k,j}; O) = &\max_{e:c_{k,j}(e)=1}{}^* \{\alpha_{k-1}[s^S(e)] + \sum_{\substack{i=1\\ i\neq j}}^{q_2} c_{k,i}(e)\lambda_k[C_{k,i}; I] + \beta_k[s^E(e)]\}\\ &- \max_{e:c_{k,j}(e)=0}{}^* \{\alpha_{k-1}[s^S(e)] + \sum_{\substack{i=1\\ i\neq j}}^{q_2} c_{k,i}(e)\lambda_k[C_{k,i}; I] + \beta_k[s^E(e)]\}\end{aligned}$$

with initial values, $\alpha_0(s) = 0$, if $s = 0$ and $\alpha_0(s) = -\infty$, if $s \neq 0$, and $\beta_n(s) = 0$, if $s = 0$ and $\beta_n(s) = -\infty$, if $s \neq 0$, where h_{α_k} and h_{β_k} are normalization constants which, in the hardware implementation of the SISO, are used to prevent the buffer overflow.

The final decision is obtained from the bit reliability computation of $U_{k,j}$; $j = 1, 2, \cdots, p_2$, passing it through a hard limiter, as

$$\lambda_k(U_{k,j};\, O) = \max^*_{e:u_{k,j}(e)=1} \{\alpha_{k-1}[s^S(e)] + \sum_{i=1}^{q_2} c_{k,i}(e)\lambda_k[C_{k,i};\, I] + \beta_k[s^E(e)]\} - \max^*_{e:u_{k,j}(e)=0} \{\alpha_{k-1}[s^S(e)] + \sum_{i=1}^{q_2} c_{k,i}(e)\lambda_k[C_{k,i};\, I] + \beta_k[s^E(e)]\}$$

The outer SISO accepts the extrinsics from the inner SISO as input reliabilities of coded bits of the outer encoder. For the outer SISO there is no external observation from the channel. The outer SISO uses the input reliabilities for calculation of new extrinsics $\lambda_k(C_{k,j};\, O)$ for coded bits. These are then provided to the inner SISO module.

4.4 Structure of Iterative Decoder for Punctured Outer Code

The structure of iterative decoder for punctured outer code is shown in Fig. 16

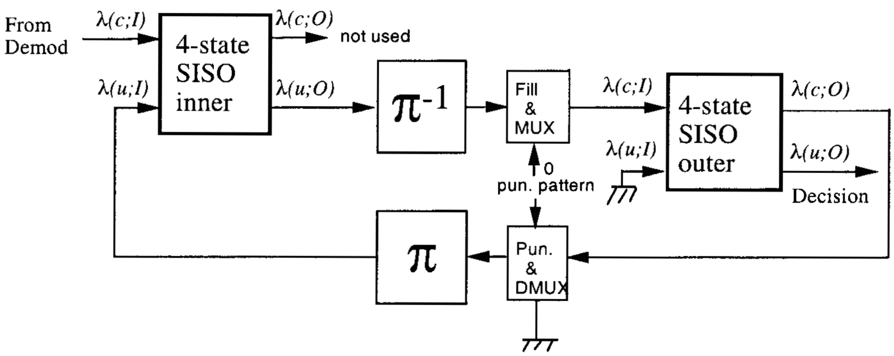

Figure 16: Turbo decoder low complexity SCTCM

4.5 Complexity of proposed punctured SCTCM with suboptimum iterative decoding

Reduction in complexity of SCTCM encoder is already discussed. We also intend to reduce the complexity of the iterative decoder. For example in the SISO algorithm if we don't use correction terms we can speed up the decoder by paying a very small penalty (about 0.2 dB) in performance by proper scaling. All operations in this case are similar to those of a Viterbi decoder, namely Add-Compare-Select operations.

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